Market Risk

Estimating Market Risk Measures: An Introduction & Overview

1. Profit or loss data = $P_n + CF - P_0$
2. Arithmetic Return = $\frac{P_n - P_0 + CF}{P_0}$
3. Geometric Return = $\ln\left(\frac{P_n + CF}{P_0}\right)$
4. VaR: $[(\alpha \times n) + 1]$th observation
5. Delta-normal VaR:

   $VaR(\alpha\%) = (\mu_r - \sigma_r \times z_\alpha)$

   (In % terms)

   $VaR(\alpha\%) = P_0 \times (\mu_r - \sigma_r \times z_\alpha)$

   (In $ terms)

6. Weights in Expected Shortfall = $\left(\frac{1}{1 - confidence\ Level}\right)$
7. Lognormal VaR:

   $VaR(\alpha\%) = P_0 \times (1 - e^{\mu_r - \sigma_r \times z_\alpha})$

8. $Se(q) = \frac{\sqrt{p(1-p)/n}}{f(q)}$
9. $SE_{Quantile} = \sqrt{Variance_{Quantile}}$
10. Confidence Interval for VaR:

    $q + se(q) \times z_\alpha > VaR > q - se(q) \times z_\alpha$

Non-Parametric Approaches

1. Age – weighted Historical Simulation:

   $W(i) = \frac{\lambda^{i-1}(1-\lambda)}{1-\lambda^n}$

2. Volatility weighted Historical Simulation:

   $r^* = \left(\frac{\sigma_{T,i}}{\sigma_{t,i}}\right) r_{t,i}$

Parametric Approaches (II): Extreme Value

1. GEV distribution:

$$F(X|\xi,\mu,\sigma) = \exp\left[-\left(1+\xi\times\frac{x-\mu}{\sigma}\right)^{-\frac{1}{\xi}}\right] \text{ if } \xi \neq 0$$

$$F(X|\xi,\mu,\sigma) = \exp\left[-\exp\left(\frac{x-\mu}{\sigma}\right)\right] \text{ if } \xi = 0$$

2. Generalized Pareto Distribution:

$$1-\left[1+\frac{\xi x}{\beta}\right]^{-\frac{1}{\xi}} \text{ if } \xi \neq 0$$

$$1-\exp\left[-\frac{x}{\beta}\right] \text{ if } \xi = 0$$

3. POT:

$F_u(x) = P\{X - u \leq x \mid X > u\} = \frac{F(x+u)-F(u)}{1-F(u)}$

4. VaR using POT:

$u + \frac{\beta}{\xi}\left[\left[\frac{n}{N_u}(1-confidence\ level)\right]^{-\xi} - 1\right]$

5. ES: $\frac{VaR}{1-\xi} + \frac{\beta-\xi U}{1-\xi}$

Backtesting VaR

1. Model Accuracy Test:

$$z = \frac{x-pT}{\sqrt{p(1-p)\times T}}$$

2. H_0 : Model = Unbiased

   H_a : Model ≠ Unbiased

3. $L.R_{CC} = L.R_{UC} + L.R_{ind}$
4. Log-likelihood ratio:

   $LR_{UC} = -2\ln\left[(1-P)^{T-N}P^N\right] + 2\ln\left\{[(1-N/T)]^{T-N}P^N\right\}$

5. Probability of exception : p = 1 – c
6. Failure rate = $\frac{N}{T}$
7. No. of exception = (1 – p) × T

VaR MAPPING

1. $R_i = \alpha_i + \beta_i R_M + \varepsilon_i$
2. $R_P = \sum_{i=1}^{N} w_i R_i = \sum_{i=1}^{N} w_i \beta_i R_M + \sum_{i=1}^{N} w_i \varepsilon_i$
3. $\beta_P = \sum_{i=1}^{N} w_i \beta_i$
4. $V(R_P) = \beta_P^2 \times \sigma_M^2 + \sum_{i=1}^{N} w_i^2 \times \sigma_{\varepsilon,i}^2$
5. $Undiversified\ VaR = \sum_{i=1}^{N} |x_i| \times V_i$
6. $Diversified\ VaR = \alpha\sqrt{x' \sum x} = \sqrt{(x \times V)' R (x \times V)}$
7. $Tracking\ Error\ VaR = \alpha\sqrt{(x - x_0)' \sum (x - x_0)}$
8. $Variance\ improvement = 1 - (tracking\ error/benchmark\ VaR)^2$
9. $Forward_t = (F_t - K)e^{-rt}$
10. $One - day\ risk\ horizon: -z\alpha_{/2}\sigma\sqrt{T}$

Empirical Properties of Correlations: How do Correlations Behave in the Real World?

1. Mean Reversion Rate:

   St - St-1 =a (μ - S t-1) $\Delta t + \sigma_S \varepsilon \sqrt{\Delta t}$

2. Auto Correlation:

   $AC_{(\rho_t, \rho_{t-1})} = \frac{Cov_{(\rho_t, \rho_{t-1})}}{\sigma(\rho_t) \times \sigma(\rho_{t-1})}$

Financial Correlation Modeling – Bottom-Up Approaches

1. Correlation copula:
   $C\,[G_1(u_1),\ldots,G_n(u_n)] = F_n[F_1^{-1}(G_1(u_1)),\ldots,F_n^{-1}(G_n(u_n));\rho_F]$
2. The Gaussian default time copula:
   $C_{GD}\,[Q_i(t),\ldots,Q_n(t)] = M_n[N_1^{-1}(Q_1(t)),\ldots,N_n^{-1}(Q_n(t));\rho_M]$
3. The Gaussian copula for the bivariate standard normal distribution:
   $C_{GD}\,[Q_B(t),\ldots,Q_C(t)] = M_2[N_1^{-1}(Q_B(t)),\ldots,N_n^{-1}(Q_C(t));\rho]$
4. $M^n(\blacksquare) = Q_i(\tau_i)$

Empirical Approaches to Risk Metrics and Hedging

1. Calculate the regression hedge adjustment factor, beta:

$\Delta y_t^N = \alpha + \beta\Delta y_t^R + \varepsilon_t$

2. $F^R = F^N \times \left(\frac{DV01^N}{DV01^R}\right) \times \beta$

3. Two variables regression Hedge: (For a combination of 10-and 30-year swap)

$\Delta y_t^{20} = \alpha + \beta^{10}\Delta y_t^{10} + \beta^{30}\Delta y_t^{30} + \varepsilon_t$

4. Regressing change on change

$\Delta y_t = \alpha + \beta\Delta x_t + \Delta\varepsilon_t$

5. Nominal on real (not change)

$y_t = \alpha + \beta x_t + \varepsilon_t$

6. $\varepsilon_t = \rho\varepsilon_{t-1} + v_t$

The Evolution of Short Rates and the Shape of the Term Structure

1. 2-year Short Rate:
   $\hat{r}(2) = \sqrt[2]{(1+r_1)(1+r_2)}-1$
2. 3-year Short Rate:
   $\hat{r}(3) = \sqrt[3]{(1+r_1)(1+r_2)(1+r_3)}-1$
3. Jensen's inequality:
   $E\left[\frac{1}{1+r}\right] > \frac{1}{E(1+r)}$

Volatility Smiles

1. Put - Call Parity

   $P_{mk} - p_{BSM} = c_{mkt} - c_{BSM}$

   $c - p = S - PV(x)$

   Or,

   $c - p = S - X.e^{-rT}$ or $PV(X) = X\,e^{-rT}$

Credit Risk

Portfolio Credit Risk

1. $\rho_{1,2} = \frac{\pi_{12} - \pi_1 \pi_2}{\sqrt{\pi_1(1-\pi_1)}\sqrt{\pi_2(1-\pi_2)}}$
2. Conditional Cumulative Default Probability:

   $P(m) = \Phi\left(\frac{k_i - \beta_i \bar{m}}{\sqrt{1-\beta_i^2}}\right)$
3. Single Factor Model:

   $a_i - \beta_i \bar{m} = \sqrt{1-\beta_i^2}\,\varepsilon_i$
4. $\rho = \frac{\Phi\binom{k}{k} - \pi^2}{\pi(1-\pi)}$
5. $\rho_{1,2} = \frac{\Phi\binom{ki}{kj} - \pi_1 \pi_2}{\sqrt{\pi_i(1-\pi_i)}\sqrt{\pi_j(1-\pi_j)}}$
6. $X(m) = P(m) = \Phi\left(\frac{k - \beta \bar{m}}{\sqrt{1-\beta^2}}\right)$
7. $\Phi^{-1}(\bar{x}) = \left(\frac{k - \beta \bar{m}}{\sqrt{1-\beta^2}}\right)$

Credit And Debt Value Adjustment

1. $C.V.A = \text{LGD} \times \sum_{i=1}^{m} d(ti) \times EE(t_i) \times PD(t_{i-1,t_i})$
2. CVA as a spread:

   $\frac{CVA(t,T)}{CDS_{premium}(t,T)} = X^{CDS} \times EPE$
3. $V(i) = \Delta CVA_{NS,i} = CVA(NS,i) - CVA(NS)$
4. BCVA = CVA + DVA

   $CVA = +LGD_c \times \sum_{i=1}^{m} EE(t_i) \times PD_C(t_{i-1,t_i})$

   $DVA = -LGD_1 \times \sum_{i=1}^{m} NEE(t_i) \times PD_1(t_{i-1,t_i})$
5. BCVA Spread:

   $\frac{BCVA(t,T)}{CDS_{premium}(t,T)} = X_C^{CDS} \times EPE - X_1^{CDS} \times ENE$

The Evolution of Stress Testing Counterparty Exposures

1. Loan Portfolios:

$$EL = \sum_{i-1}^{N} PD_i \times EAD_i \times LGD_i$$

$$EL_S = \sum_{i-1}^{N} PD_i^S \times EAD_i \times LGD_i$$

2. Derivatives Portfolios:

$$EL = \sum_{i-1}^{N} PD_i \times [EPE_i \times \alpha] \times LGD_i$$

$$EL_S = \sum_{i-1}^{N} PD_i^S \times [EPE_i^s \times \alpha] EAD_i \times LGD_i$$

3. $CVA_n = LGD_n \times \sum_{i=1}^{T} EE_n(t_j) \times PD^*(t_{j-1,} t_j)$

4. $CVA_s = \sum_{n=1}^{n} LGD_n^* \times \sum_{j=1}^{T} EE_n^s(t_j) \times PD_n^s(t_{j-1}, t_j)$

5. $BCVA = +\sum_{n=1}^{n} LGD_n^* \times \sum_{j=1}^{T} EE_n^*(t_j) \times PD^*(t_{j-1}, t_j) \times S_I{}^*(t_{j-1})$

$$-\sum_{n=1}^{n} LGD_I^* \times \sum_{j=1}^{T} NEE_n^*(t_j) \times PD^*(t_{j-1}, t_j) \times S_n{}^*(t_{j-1})$$

Structured Credit Risk

1. Loan interest:
$$\left(N - \sum_{t=1}^{T} d_t\right) \times (LIBOR + spread) \times par$$
2. Proceeds (par) from redemption of surviving loans:
$$\left(N - \sum_{t=1}^{T} d_t\right) \times par$$
3. Recovery in final year:
$$R_T = 0.4 d_T \times loan\ amount$$
4. Residual in trust account
$$\sum_{\tau=1}^{T} (1+r)^{t-\tau} OC_t$$

An Introduction to Securitization

1. WAL = $\sum \left(\frac{a}{365}\right) \times PF(t)$
2. CPR = 1-(1-SMM)^{12}
3. SMM = 1-(1-CPR)$^{1/12}$

Operational Risk

Risk Capital Attribution and Risk Adjusted Performance Measurement

1. Economic capital = risk capital + strategic risk capital
2. RAROC = $\frac{\text{after tax expected risk−adjusted net income}}{\text{economic capital}}$;

   $\frac{expected\ rebenues - costs - expected\ losses - taxes + return\ on\ economic\ capital \pm transfer}{economic\ capital}$
3. Hurdle Rate :

   $h_{AT} = \frac{[(CE \times R_{CE}) + (PE \times R_{PE})]}{(CE + PE)}$
4. Adjusted RAROC = RAROC – $\beta_E (R_M - R_F)$
5. $R_{CE} = R_f + \beta_{CE} (R_M - R_F)$

Basel I, Basel II, and Solvency II

1. Credit Equivalent Amount: Max (V,0) + (a x L)
2. Total RWA:

   $\sum_{i=1}^{N} w_i L_i + \sum_{j=1}^{M} w_j C_j$
3. Market Risk Capital Requirement: Max (VaR_{t-1} , m_c x VaR_{avg}) + SRC
4. Total Capital= 0.08 × (credit risk RWA + market risk RWA)
5. Market RWA = $12 \cdot 5 \times \left(max(VaR_t, m_c \times VaR_{avg})SRC\right)$
6. Credit RWA = $\sum(RWA\ on - balance\ sheet\ ) + \sum(RWA\ off - balance\ sheet\ )$
7. $VaR_{99.9\%,1-year} \approx \sum_i EAD_i \times LGD_i \times WCDR_i$
8. Expected loss:

   EL = $\sum_i EAD_i \times LGD_i \times PD_i$
9. Required Capital = EAD_i x LGD_i x ($WCDR_i$ -PD_i)
10. $\rho = 0.12 \times (1 + e^{-50 \times PD})$
11. From a counterparty's perspective, the capital required for the counterparty incorporates a maturity adjustment as follows:

    Required Capital = EAD x LGD x (WCDR – PD) x MA

    Where MA = Maturity Adjustment; MA = $\frac{(1+(M-2\cdot 5)\times b)}{(1-1\cdot 5\times b)}$; M = Maturity of the exposure
12. RWA = $12 \cdot 5 \times [EAD \times LGD \times (WCDR) \times MA\ ]$
13. Minimum Capital Requirements:

    Total capital=0.08 × (credit risk RWA + market risk RWA + operational risk RWA)

Basel II, Basel III, and other Post-Crisis Changes

1. Stressed VaR: max(VaR_{t-1},m_c × VaR_{avg}) + max($SVaR_{t-1}$,m_s × $SVaR_{avg}$)
2. Liquidity Coverage Ratio = $\frac{high\ quality\ liquid\ asset}{net\ cash\ outflows\ in\ a\ 30-day\ period}$ ≥100%
3. Net stable funding ratio = NSFR =$\frac{amount\ of\ available\ stable\ funding}{amount\ of\ required\ stable\ funding}$ ≥100 %

Liquidity Risk

Liquidity and Leverage

1. Leverage ratio:

   $L = \frac{A}{E} = \frac{(E+D)}{E} = 1 + \frac{D}{E}$

2. Leverage effect:

   $r_E = Lr_A - (L-1)r_D$

   ROE = (leverage ratio × ROA) – [(leverage ratio – 1) × cost of debt]

3. Effect of increasing leverage:

   $\frac{\delta r_E}{\delta L} = r_A - r_D$

4. Transaction cost (99%confidence interval): +/- P × $\frac{1}{2}$ (s + $2.33\sigma_s$)

   Spread risk factor: ½ (s + $2.33\sigma_s$)

5. 1 day position VaR:

   $VaR_t \times \sqrt{T}$

6. VaR when position can be liquidated for a period of days: $VaR_t \times \sqrt{\frac{(1+T)(1+2T)}{6T}}$

Investment Risk

Factor Theory

1. Investor Risk Premium:

   $E(R_M) - R_F = \bar{\gamma} \times \sigma_m^2$

2. Security Market line: $E(R_i) - R_F = \frac{cov(R_I, R_M)}{var(R_M)} \times [E(R_M) - R_F] = B_i \times [E(R_M) - (R_F)]$

3. Multifactor Model:

   $M = a + b_1 f_1 + b_2 f_2 + \cdots + b_k f_k$

4. SDF model:

   $P_i = E[m \times payoff_i]$

5. $E(R_i) - R_F = \frac{cov(R_{i,m})}{var(m)} \times \left(\frac{-var(m)}{E(m)}\right) = \beta_{i,m} \times \lambda_m$

6. $E(R_i) = R_F + \beta_{i,1} \times E(f_1) + \beta_{i,2} \times E(f_2) + \cdots + \beta_{i,k} \times E(f_k)$

Factors

1. $E(R_M) - R_F = \bar{\gamma} \times \sigma_m^2$

2. Fama – French three-factor Model:

   $E(R_i) = R_F + \beta_{i,Mkt} \times E(R_m - R_F) + \beta_{i,SMB} \times E(SMB) + \beta_{i,HML} \times E(HML)$

3. Fama-French Model with Momentum Effect:

   $E(R_i) = R_F + \beta_{i,Mkt} \times E(R_m - R_F) + \beta_{i,SMB} \times E(SMB) + \beta_{i,HML} \times E(HML) + \beta_{i,WML} \times E(WML)$

Alpha (and the Low-Risk Anomaly)

1. $R_t{}^{ex} = R_t - R_t{}^B$
2. $\alpha = \frac{1}{T}\sum_{t=1}^{T} R_t{}^{ex}$
3. Fundamental Law of Active Management: IR = $\frac{\alpha}{\sigma}$
4. $\alpha = R_t - R_F$
5. Sharpe ratio = $\frac{\bar{\bar{R}}_t - \bar{R}_F}{\sigma}$
6. IR$\approx \text{IC} \times \sqrt{\text{BR}}$
7. E (R_i) = R_F + B$[E(R_M) - R_f]$
8. Fama and French three factor model = $R_i - R_F$ = α + $\beta_{i,Mkt} \times (R_M - R_F)$ +$\beta_{i,SMB}$ $\times$ (SMB) +$\beta_{i,HML} \times$ (HML)

Portfolio Construction

1. Alpha = volatility × (information coefficient) × (score)
2. Risk Aversion = $\frac{\text{information ratio}}{2\times\text{active risk}}$
3. Average alpha for the stocks with forecasts

   = $\frac{(\textit{weighted average of the alphas with forecast})}{(\textit{value}-\textit{weighted fracttion of stocks with forcasts})}$

4. Marginal contribution to value added = (alpha of asset) – [2 × (risk aversion) × (active risk) × (marginal contribution to active risk of asset)]
5. Cost : – (cost of selling) < (marginal contribution to value added) < (cost of purchase)
6. Range : [2 × (risk aversion)) × (active risk) × (marginal contribution to active risk)] – (cost of selling) < alpha of asset < [2 × (risk aversion)) × (active risk) × (marginal contribution to active risk)] + (cost of purchase)
7. Portfolio construction technique:

   (Portfolio alpha) - (risk aversion) × (active risk)^2 – (transaction cost)

Portfolio Risk: Analytical Methods

1. Diversified VaR: $VaR_P = Z_C \times \sigma_P \times P$
2. Individual VaR: $VaR_P = Z_C \times \sigma_i \times |P| = Z_C \times \sigma_i \times |w_i| \times P$
3. Standard deviation of a two-asset portfolio:

   $$\sigma_P = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho_{1,2}\sigma_1\sigma_2}$$

4. VaR of a two-asset portfolio: $VaR_P = Z_c P\sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho_{1,2}\sigma_1\sigma_2}$
5. Undiversified VaR: $VaR_P = \sqrt{VaR_1^2 + VaR_2^2 + 2VaR_1VaR_2} = VaR_1 + VaR_2$
6. Marginal VaR: $MVaR_i = \frac{VaR}{\text{portfolio value}} \times \beta_i$ or $Z_c \frac{cov(R_i, R_p)}{\sigma_p}$
7. Component VaR:

   $CVaR_i = (MVaR_i) \times (w_i \times P) = VaR \times \beta_i \times w_i$
8. σ_P of 2 asset portfolio with equal weights and same s.d and same correlations between each pair:

   $$\sigma_P = \sigma \times \sqrt{\frac{1}{N} + \left(1 - \frac{1}{N}\right)\rho}$$

VaR and Risk Budgeting in Investment Management

1. Surplus = Assets - Liabilities
2. ΔSurplus = ΔAssets - ΔLiabilities
3. Return on the surplus:

   $$R_{surplus} = \frac{\Delta Surplus}{Assets} = \frac{\Delta Assets}{Assets} - \left(\frac{\Delta Liabilities}{Liabilities}\right)\left(\frac{Liabilities}{Assets}\right) = R_{asset} - R_{liabilities}\left(\frac{Liabilities}{Assets}\right)$$

4. Weight of portfolio managed by manager i= $\frac{IR_i \times (\text{portfolio's tracking error})}{IR_P(\text{manager's tracking error})}$

Risk Monitoring and Performance Measurement

1. Liquidity Duration:

   LD = $\frac{Q}{(0.10 \times V)}$

Portfolio Performance Evaluation

1. Sharpe Ratio: $S_A = \left[\frac{\bar{R}_A - \bar{R}_F}{\sigma_A}\right]$
2. Treynor Measure: $T_A = \left[\frac{\bar{R}_A - \bar{R}_F}{\beta_A}\right]$
3. Jensen's Alpha: $\alpha_A = R_A - E(R_A)$
4. Information Ratio: $IR_A = \left[\frac{\bar{R}_A - \bar{R}_B}{\sigma_{A-B}}\right]$
5. $M^2 = R_P - R_M$
6. Null (H_0): True alpha is zero

   Alternate (H_A) : True alpha is not zero
7. Statistical significance of alpha returns: $t = \frac{\alpha - 0}{\sigma/\sqrt{N}}$
8. Measuring Market Timing with Regression:

   $R_P - R_F = \alpha + \beta_P(R_M - R_F) + M_P(R_M - R_F)D + \varepsilon_P$
9. Measuring Market Timing with a Call Option Model:

   100% invested in the S & P 500 if $E(R_M) > R_F$

   100% invested in Treasury bills if $E(R_M) < R_F$
10. Asset Allocation Attribution:

    $[b_1 R_{B1} + b_2 R_{B2} + \cdots + b_n R_{Bn}] - R_B$

    $R_P - [b_1 R_{B1} + b_2 R_{B2} + \cdots + b_n R_{Bn}]$